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MODELING OF VORTEX STRUCTURES ON THE LATERAL SURFACE OF A CYLINDER IN A LONGITUDINAL FLOW

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The problem of generation of vortex structures by ejection of a jet through an annular slot on a cylindrical surface in a longitudinal flow is considered. Results of modeling of large-scale vortices and their effect on local characteristics of the wall boundary layer are presented. The modeling is based on numerical integration of the Navier-Stokes equations in an axisymmetric formulation. An unsteady-state flow with separation and entrainment of vortices by the main stream is obtained. The decrease in the friction drag achievable in devices of this kind is estimated.

Introduction. The basic difficulty in theoretical and experimental studies of turbulence is the presence of various types of vortices in the flow, from large-scale vortices to the finest ones, which decay immediately due to the effect of viscous dissipation. As a result, even for moderate Reynolds numbers the interval of the scales requiring resolution can barely be achieved both in experimental studies and in numerical modeling. Nevertheless, processes of generation, stability, and interaction of vortices of various scales are of great interest. In spite of the fact that the small-scale part of the spectrum is responsible mainly for dissipation processess, it can also generate or supply energy to large-scale structures owing to local instability.

The effect of vortex structures located in the wall boundary layer (WBL) on the flow pattern and, in particular, on the friction drag is substantial. It is known that generation of large-scale vortices in the WBL can result in a decrease in the friction drag over the whole surface. Another field of application of vortex structures is control of separation of the WBL [2]. The problem of the effect of ordered large-scale vortices on the turbulence structure in the WBL is no doubt of interest as well.

All the above problems stimulate the study of vortex structures concerning their generation, interaction, and stability. Thus, in [3] the problem of vortex generation in a WBL was solved with a fin placed across the flow.

The present authors investigate vortex generation by ejection of a liquid jet. A longitudinal flow around a cylinder is considered. For generation of vortices a narrow ($D/\Delta = 100$) angular slot is made on the cylindrical surface and fluid flows out through this slot into the main stream at a right angle. The fluid is assumed to be viscous and incompressible and the flow, two-dimensional and laminar. The object of the study is the Navier-Stokes equations written in terms of velocity-pressure variables in cylindrical coordinates. The method of investigation is numerical integration of these equations.

Choice of Scales, the System of Equations, and Boundary Conditions. The chosen characteristic scales of the quantitites are used to make the equations dimensionless. Two geometrical dimensions (the diameter D of the cylinder and the width Δ of the slot) appear in the problem. In the case of external (longitudinal or transverse) flow around the cylinder, the diameter is used, as a rule, as the geometrical scale. However, in the present study the problem of vortex formation in the direct vicinity of the slot is considered. Inhomogeneities arising in the liquid flow are caused by the outflowing jet and have transverse sizes of the order of Δ for any $D \gg \Delta$. Moreover, at $D \to \infty$, the problem reduces to vortex formation on a plane, where the width of the slot is the only geometrical

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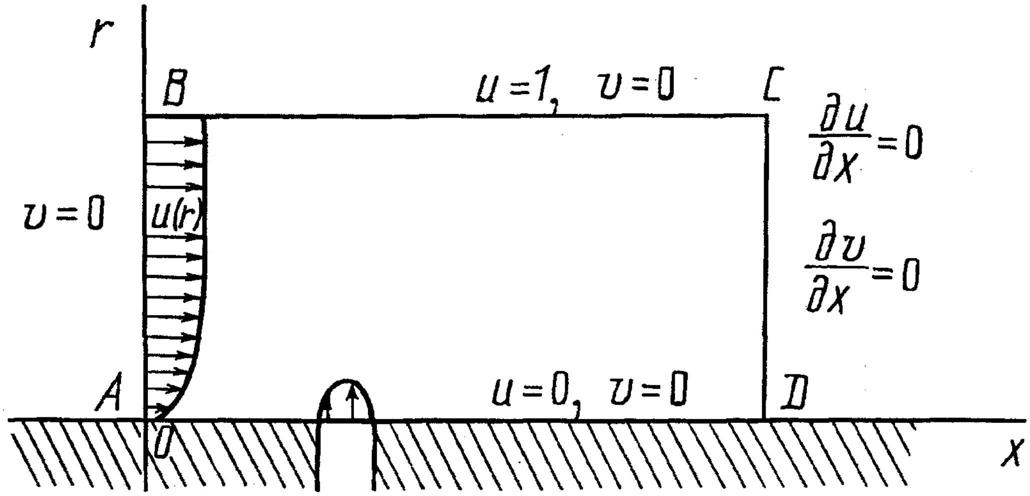

Fig. 1. Computation region.

parameter. In view of this the slot width Δ is chosen as a characteristic scale of length. The main-stream velocity U_∞ is taken as a velocity scale. Thus, the Reynolds number is expressed as

$$\mathrm{Re}_\Delta = U_\infty \Delta / \nu \,. \tag{1}$$

It should be noted that the problem contains two more dimensionless parameters:

$$q = U_j / U_\infty \,, \tag{2}$$

$$\delta = D / \Delta \,. \tag{3}$$

Combining (1), (2), and (3), it is possible to obtain other dimensionless combinations, for example,

$$\mathrm{Re}_j = q\,\mathrm{Re}_\Delta = U_j \Delta / \nu \tag{4}$$

which is the Reynolds number related to the angular jet flowing out into the main stream. Similarly to (1), it can characterize the local stability of the jet, and in the limiting case $U_\infty = 0$, $D \to \infty$, the problem assumes the form of a plane submerged jet flowing out into a semiinfinite space. In this case [4] critical number (4) is Re = 3.7.

The main object of interest in the present study is existence of unsteady-state separation and entrainment of vortices. Assuming that the dimensions of a vortex are of the order of Δ and its velocity is of the order of U_∞, expression (1) will be considered as a characteristic Reynolds number.

A system of dimensionless Navier-Stokes equations in cylindrical coordinates is given, for example, in [7]. In what follows, quantities that are referred to U_∞ and Δ will be indicated throughout by corresponding lower-case letters. Figure 1 illustrates the choice of the calculation region and the formulation of the boundary conditions. For the components of the velocity the following conditions are taken at the location of the slot:

$$u = 0\,, \quad v = u_j\,, \quad x \in \Delta\,; \quad u = 0\,, \quad v = 0\,, \quad x \notin \Delta\,. \tag{5}$$

Numerical Procedure. Discretization of the equations is based on the method of a control volume. Computation networks for the variables u, v, and p are shifted by a half step [5]. The scheme is implicit, and the correction to the velocity field at each time step is carried out by the SIMPLEC method [6] so as to satisfy the discontinuity condition with a preset accuracy. Convective terms are approximated using the QUICK scheme [7]. Unsteady-state terms are approximated using Peyrier's scheme of second-order accuracy. The solution procedure, the schemes used, and other aspects of the algorithm are given in [3].

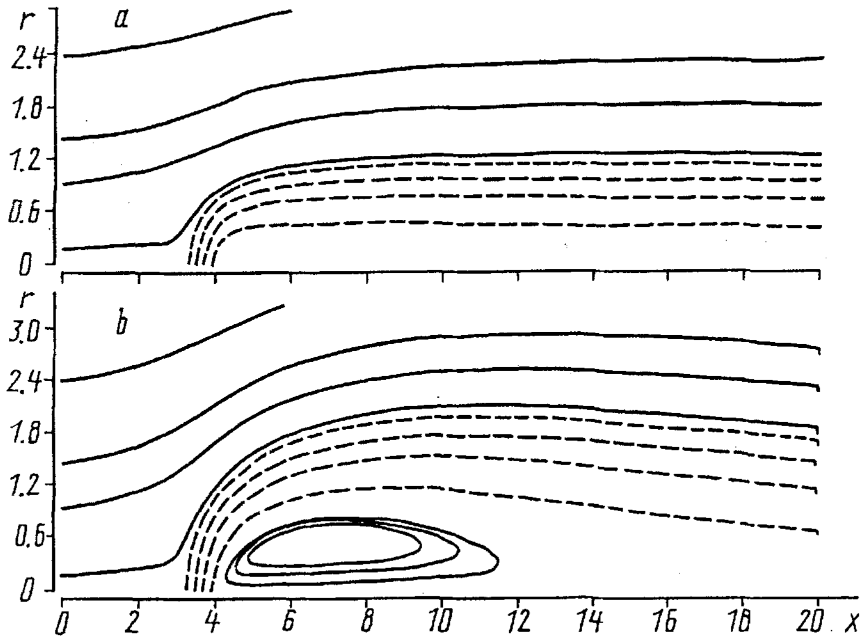

Fig. 2. Isolines of the stream function in a steady-state flow: a) $Re_\Delta = 1$, $q = 1$; b) 50 and 1.5.

A nonuniform rectangular network was chosen. It contained 60×30 nodes covering the region ABCD, shown in Fig. 1. Its size was 25×10 units (dimensionless). In the region $3 < x < 4$ on the cylindrical surface, the disposition of the annular slot, which had unit width upon normalization, was simulated using conditions (5). The minimum steps of the network were $\Delta r = 0.1$ near the wall and $\Delta x = 0.2$ in the vicinity of the slot.

For comparison with an undisturbed flow the steady-state problem for $U_j = 0$ was calculated at first in every experiment at fixed parameters Re_Δ and q. The solution was used as initial data in the unsteady-state problem disturbed by a liquid jet ejected through the annular slot.

Calculation Results. The following effects caused by a liquid jet ejected through the slot at a right angle to the main-stream direction are observed:

separation of the main stream;

formation of a wake downstream along the cylindrical surface;

an increase in the thickness of the boundary layer downstream of the slot;

a change in the distribution of the friction stress on the cylindrical surface and a change in the pressure distribution.

Depending of the value of Re_Δ and q, the flows can be divided into three types.

I. The main-stream head is the dominant force, and viscosity forces are large. In this case the outflowing jet is pressed to the cylindrical surface, not having sufficient momentum for separation downstream of the slot. The flow pattern is steady-state and is shown in Fig. 2 as isolines of the stream function. Here $Re_\Delta = 1$, $q = 1$. The distributions of the pressure and friction undergo insignificant changes relative to the undisturbed boundary layer on the cylindrical surface. A certain decrease in local friction is observed downstream of the slot. However, it should be noted that at small Reynolds numbers the values of these disturbances of the steady boundary layer are small and it is difficult to estimate them on the basis of the complete Navier-Stokes equations.

II. The second type of flow occurs when inertia forces are sufficient to separate the jet from the cylindrical surface and viscous forces cease being dominant. This type corresponds to moderate Reynolds numbers and is also steady. In this case a circulation vortex zone forms downstream. Its dimensions, growth rate, and stability depend on the parameters q and Re_λ. The corresponding flows are shown in Fig. 2b as isolines of the stream function $\psi(x, r) = C$ for $q = 1.5$ and $Re_\Delta = 50$.

Depending on the choice of the constant C the line is depicted and interpreted as follows:

for $0 < C$ (solid lines), the main stream;

TABLE 1. Calculation Results for the Steady-State Problem at Re = 50

q	Vortex dimension		I	$c_f\sqrt{\mathrm{Re}_\Delta}$	n
	vertical	horizontal			
0.2	–	–	0.20	0.20	0.18
0.4	–	–	0.40	0.15	0.38
0.6	0.2	1.5	0.60	0.09	0.61
0.8	1	7–8	0.82	0.03	0.90
1.0	1.5	13–14	1.09	−0.04	1.17
1.5	3.0	18–20	1.94	−0.13	1.60

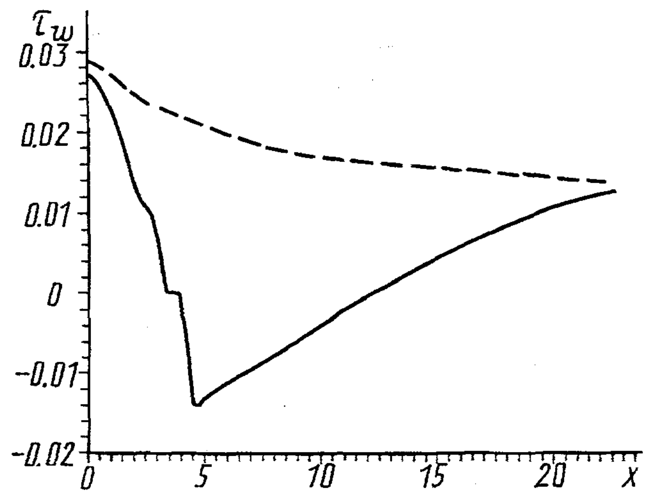

Fig. 3. Distribution of friction over the surface at $\mathrm{Re}_\Delta = 50$ and $q = 1.5$.

for $-q < C < 0$ (dashed lines), the jet flow;

for $C < -q$ (solid lines), the circulation vortex zone.

In Fig. 2 the scale along the ordinate is increased for clarity. With a circulation zone the friction distribution $f(x)$ on the cylindrical surface is nonmonotonic. The function has several local minima that are related to the formation of regions of circulation vortices of different intensities. As an example, Fig. 3 shows the function $f(x)$ corresponding to the experimental results given in Fig. 2b. It is seen that at $x > 4$ (i.e., behind the slot) a substantial portion of the function is located below the abscissa since the flow in the corresponding region is retrogressive.

Another circulation vortex zone is located upstream of the slot. Its dimensions and intensity are substantially smaller than those of the main region. The flow in it is similar to that in the vortex zone in front of a solid obstacle located in a flow. In Fig. 3 the dashed line shows the friction in a boundary layer undisturbed by a jet.

Table 1 contains calculation results on the vortex of the vortex structure as a function of the dimensionless velocity q of the liquid flowing out through the slot. In this case the horizontal dimension was determined by the point where the zero isoline of the stream function joins the cylindrical surface. This quantity can be interpreted as the zone directly affected by the vortex structure.

The vortex intensity I was determined by the relation $I = -\min \psi(x, r)$, and in this experiment $c_f\sqrt{\mathrm{Re}_\Delta}$ was 0.24. As can be seen from Table 1, the friction decreases substantially as q increases. However, this effect is caused by an increase in the size of the vortex, and this should induce an increase in the pressure resistance. Therefore, the following type of flow is the most interesting in practice.

III. The third type of flow can occur only when the jet is separated from the cylindrical surface and is unsteady. This case is characterized by large Reynolds numbers Re. The following description is illustrated by a calculation carried out for a problem with $\mathrm{Re}_\Delta = 500$, $q = 1$.

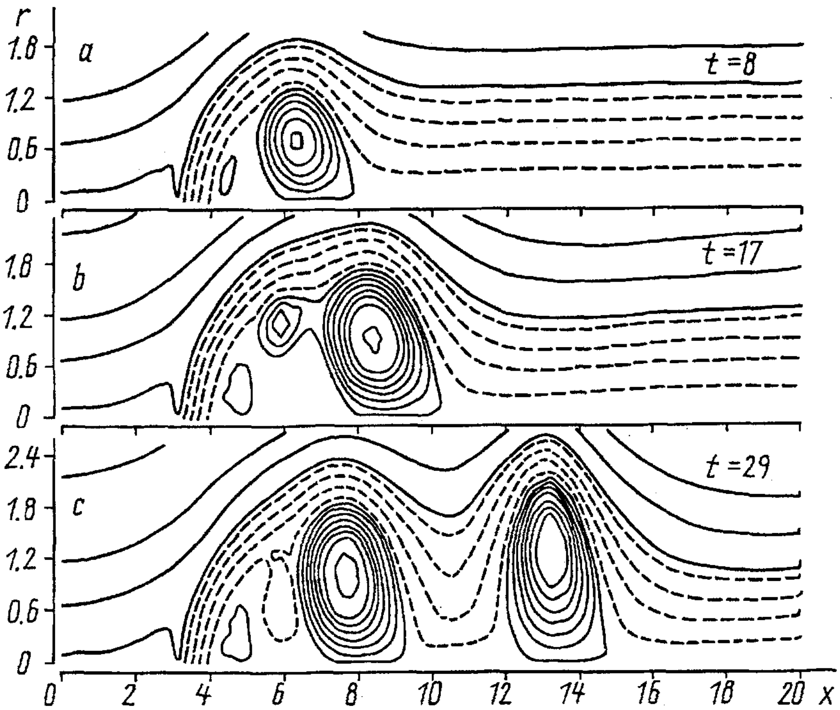

Fig. 4. Isolines of the stream function in an unsteady-state flow at different times at $\mathrm{Re}_\Delta = 500$ and $q = 1$.

Because of the internal hydrodynamic instability exhibited here, the solution of the problem is unsteady and, strictly speaking, aperiodic. The behavior of the solution displays the following regular traits:

a) at first, the vortex grows in size and moves slightly from the slot exit (see Fig. 4a);

b) then, unsteady generation of small vortices starts directly at the slot; they are carried away to the main vortex, which absorbs them. This is accompanied by some disturbances of the shape, intensity, and coordinates of its center, leading to an increase in the instability of the vortex structure (see Fig. 4b);

c) finally, separation or decay of the vortex takes place and the whole vortex or a portion of it is separated from the slot and entrained downstream along the cylindrical surface by the main stream with gradual dissipation under the action of viscous forces (see Fig. 4c).

In Fig. 5 it is shown that the behavior of the vortices is unsteady in the calculation region. Here the abscissa is the dimensionless time and the ordinate is the local minima of the stream function

$$I(t) = -\min \psi(x, r, t).$$

For clarity different vortices are reprosented by different symbols and large-scale vortices are linked by a continuous curve.

As was shown in Fig. 4c, several vortices can exist simultaneously in a region. They can be generated or destroyed by viscous dissipation or absorb one other. However, in spite of the complex nature of the behavior of the functions shown in Fig. 5, it is clearly seen that large-scale structures are present.

The vortices grow in the immediate vicinity of the slot exit, and the intensity drops after their separation during motion along the cylindrical surface due to the effect of viscous forces. Then, the next vortex is generated and starts growing, and so on.

Results of the numerical experiment can be used to estimate the dimensionless frequency of vortex decay

$$\omega = 1/\Delta t,$$

where Δt is the period. It is evident that to increase the reliability of a statistical estimate from data of numerical experiment a large number of decaying vortices should be modeled, which is expensive since it requires much

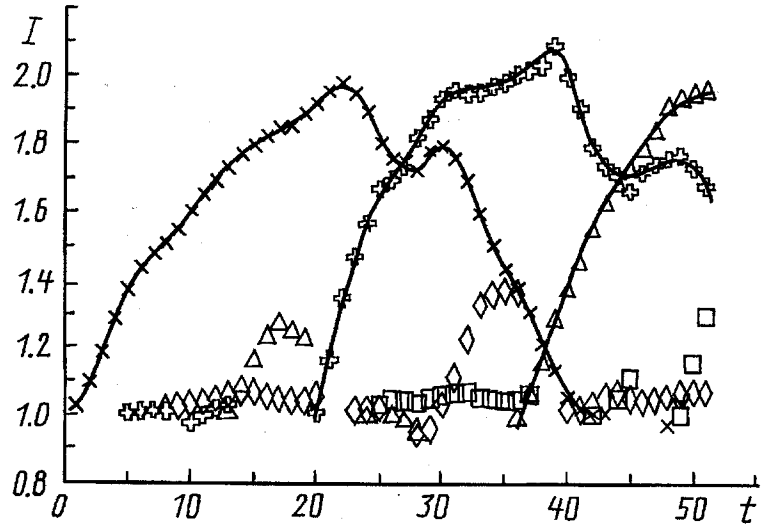

Fig. 5. Time evolution of the quantity $I = -\min \psi(x, r, t)$ at $\mathrm{Re}_\Delta = 500$ and $q = 1$.

computer time. In the above experiment the dimensionless frequency is estimated as 1/20, corresponding to Ω = 500 Hz at $\Delta = 0.001$ m, $U_\infty = 10$ m/sec.

Since in this case c_f is no longer constant but is a function of time t, friction was calculated as the time average from the beginning of ejection to the time t:

$$\bar{c} = \frac{1}{t - t_0} \int c_f(t)\, dt\,.$$

A calculation showed that in a fairly short time $\bar{c}$ becomes constant so that $\bar{c}\sqrt{\mathrm{Re}_\Delta} = 0.034$. For this flow $c_0\sqrt{\mathrm{Re}_\Delta} = 0.187$ and consequently the efficiency of the decrease in friction n is 82% in the calculation region.

Discussion and Conclusions. The present numerical study of a cylinder in a longitudinal flow with vortex structures generated by ejection through slots on the cylindrical surface was restricted to the area on this surface located in the immediate vicinity of the slot. We were compelled to restrict the calculation region. Adequate resolution of the vortex structure required an extremely dense calculation network near the slot, and extension of the network over a large distance along the cylindrical surface was very difficult because of a manifold increase in the computation time. Suffice it to say that computation of just one version of the flow up to the moment of unsteady separation of as few as five or six separate vortex structures took about 12 h of machine time on an IBM PC AT/386 computer.

In fact, because of the restriction of the computation region, in this problem only the friction in a relatively narrow region with a length of 25Δ was estimated quantitatively. Nevertheless, the present results allow the following conclusions.

1. The statement of the problem described above based on integration of the Navier-Stokes equations and the algorithm suggested for solution of the problem allow numerical investigation of steady and unsteady vortex formation on the lateral surface of a long body of revolution.

2. A certain range of the parameter q and the critical Reynolds number Re exist such that at $\mathrm{Re}_\Delta > \mathrm{Re}_\Delta(q)$ the flow is unsteady and self-oscillatory. The unsteadiness manifests itself as the existence of regimes of generation, growth, and separation of vortex structures developing downstream of the slot. There is no strict periodicity in separation and decay of the vortices because of nondeterministic formation of small-scale vortices near the slot and their interaction with the structure of the large-scale main vortex.

3. The unsteady process accompanied by decay of coherent vortices along the lateral surface may be of some interest concerning the decrease in local (and, consequently, global) friction on the lateral surface of a long body of revolution.

In conclusion it should be noted that the effect of ejection of a jet on the decrease in the total friction drag of a body in a flow was not investigated in the present study, since the solution of this problem is determined largely by the optimum between the decrease in the friction drag owing to the expense of the vortex generator and simultaneous growth of the pressure resistance, which can be achieved only from the solution of the problem of the total flow around a finite body of a specific shape.

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NOTATION

U_∞, main-stream velocity; U_j, velocity of the liquid ejected through the slot; D, diameter of the body; Δ, width of the slot on the cylindrical surface; $t = TU_\infty/\Delta$, dimensionless time; $\tau_w = \rho\nu(\partial U/\partial R)|_w$, friction on the cylindrical surface; $f = \tau_w/(0.5\rho U_\infty^2)$, dimensionless friction; $c_f = (1/S)\iint f dS$, friction drag coefficient; $S = 25\pi D$, area of the cylindrical surface within the computation region; $n = (c_0 - c_f)/c_0$, efficiency of the friction decrease; $c_0 = c_f$ at $U_j = 0$.

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